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*Otso Ovaskainen, Henrik Johan de Knegt, and María del Mar Delgado*
Preface

We have written this book primarily for two kinds of target audiences. The first one consists of those ecologists and evolutionary biologists who may not be very experienced with mathematical and statistical modelling yet, but who are interested in learning more about modelling approaches in order to integrate them with data. The second audience consists of those mathematicians, statisticians, and computer scientists who are interested in ecological and evolutionary questions, and thus wish to see how the mathematical, statistical, and computational tools that they already master can be applied in these fields of sciences. We have intended the book for researchers and graduate students, as well as advanced master students. We also hope it will be useful as a textbook for graduate-level courses in ecological modelling, statistical ecology, theoretical ecology, and spatial ecology.

After an introductory chapter, we devote one chapter to movement ecology, one to population ecology, one to community ecology, and one to genetics and evolutionary ecology. We have followed the same structure in each chapter. We start with a conceptual section, which provides the necessary biological background and motivates the modelling work. The next three sections present mathematical modelling approaches, followed by one section devoted to statistical approaches. We end each chapter with a perspectives section, where we summarize some of the key messages of the chapter, discuss some limitations of the approaches we have considered, and mention some alternative approaches.

We assume that the reader is familiar with the basics of calculus and probability. These are covered in good depth in the context of ecological modelling, e.g., in the books of Otto and Day (2007), Mangel (2006), and Hastings (1996). From the statistical side, we expect the reader to have basic knowledge of linear models (e.g., ANOVA and regression), covered, e.g., by the book by Grafen and Hails (2002) or that of Zuur et al. (2007). Some parts of this book utilize rather advanced mathematical and statistical methods compared with the background typical for graduate students in ecology and evolution. Therefore, we finish the book with two appendices, one related to mathematical and the other to statistical methods. In these appendices, we summarize a wide range of methods that are utilized in this book and that we expect to be useful for the reader more widely.

In a single book, it is not possible to present a comprehensive review of all the mathematical and statistical approaches developed for movement ecology, population ecology, community ecology, population genetics, and evolutionary biology.
Therefore, the selection of approaches and models we present is necessary limited, and biased by our own research interests. The reason why we cover such a broad range of topics rather than focusing on a more limited scope is that one key aim of this book is to link modelling approaches used in different fields of ecology and evolutionary biology to each other. With this aim, we have constructed the models in a stepwise manner, starting from simple baseline models, and then adding gradually new components. As one example, in Chapter 2 we start from a random walk model in homogeneous space. We then extend the model to heterogeneous space by considering how moving individuals may respond to spatial variation in their environments, after which we model movements in a highly fragmented landscape as a particular case study. In Chapter 3, we add to the movement model the births and deaths of individuals, resulting in a model that we will call the butterfly metapopulation model. As another example, in Chapter 2 we link dispersal kernels to movement models. In Chapter 3, we utilize dispersal kernels to construct a model for the ecological dynamics of sessile organisms, which we call the plant population model. In Chapter 4, we add to the plant population model competition among heterospecifics, resulting in what we call the plant community model. Finally, in Chapter 5 we extend the plant population model into an eco-evolutionary model by letting the individuals carry genes that influence the dispersal of their propagules.

While our aim is to integrate models with data, we discuss real data only in the context of few motivating examples. All the data analysed in the statistical sections have been generated by the mathematical models of the same chapter. We have decided to do so because with simulated data the reader has the full knowledge of the process that has generated the data, which would be never the case with real data. Therefore, knowing the underlying process enables one to pinpoint the potentials and pitfalls of different kinds of statistical methods more effectively than would be the case with real data.

As our starting point is movement ecology, much (though not all) of our emphasis is on the spatial aspects of ecology and evolution, especially on the interplay between environmental heterogeneity and ecological processes. In particular, each chapter has one section that applies the mathematical modelling approaches to study the consequences of habitat loss and fragmentation. In addition to presenting perspectives to ecological modelling, we thus hope to provide some insights on how habitat loss and fragmentation influence the movements of individuals, the dynamics of populations and communities, and evolutionary dynamics.

Writing this book would not have been possible without the support we have obtained from many people. First of all, working with the publisher has been as smooth as one could hope for. We would therefore like to thank Charles Godfray for taking the initiative of inviting us to write this book, Ian Sherman for guiding us with the initial planning, and Jennifer Dunne for providing feedback. Lucy Nash has worked as an excellent editor; her help has been invaluable through the writing process. For permission to reproduce copyrighted material, we would like to thank the National Academy of Sciences USA (Figures 2.2A–C and 4.2A–C), the Ecological
Society of America (Figure 2.2D), Oxford University Press (Figure 3.2CD), Nature Publishing Group (Figure 3.2E), and John Wiley and Sons (Fig. 5.2A,B,C).

Most importantly, the book has benefited greatly from comments that we have received from several colleagues. We would thus like to thank Nerea Abrego, Florian Hartig, Arild Husby, Jussi Jousimo, Etsuko Nonaka, Iñaki Odriozola, Bart Peeters, Johannes Signer, Tord Snäll and Eugenia Soroka for providing a large number of suggestions and corrections that greatly improved the contents. We thank Kaisa Torppa for helping us to compile the index, Sami Ojanen for producing the maps of Figures 2.2 and 3.2., and Juha Merilä for permission to use the photo of Fig. 5.2D. Our warmest thanks go to Ilkka Hanski, the leader of the Metapopulation Research Centre, where this book has been written. This is not only for the feedback and encouragement that he has provided during the writing process, but also and especially for sharing with us his passion for science.

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Approaches to ecological modelling

The motivation for conducting any kind of research in ecology and evolutionary biology, including modelling, is to advance knowledge about ecological and evolutionary processes. This happens mostly with small steps that refine the existing views by bringing new pieces of knowledge, but sometimes through revolutionary ideas that challenge existing paradigms. For an excellent introduction to the philosophy of science in the context of ecological research, we refer to the book by Hilborn and Mangel (1997). Briefly, the process of doing science involves the interplay between theories, hypotheses, models, and data. Theories summarize our present understanding on how we consider nature to work. They are the highest form of scientific knowledge, as they have resisted repeated testing and scrutiny through experiments and observations. Hypotheses are proposed explanations of how nature might work, requiring new data to test the validity of the proposed explanation. Theories and hypotheses can be expressed verbally, but they can also be made more specific by expressing them with the help of mathematical models. Different models of the same phenomena can be viewed as alternative hypotheses, and alternative parameter values of those models can be seen as even more specific hypotheses. Data are essential for examining the extent to which hypotheses are supported. This can be done by examining whether a single hypothesis is supported by data, or by quantifying the degree of support for alternative hypotheses simultaneously. Observing a contradiction between a hypothesis and data is a key step for making scientific progress, as it makes one think why nature does not work as was hypothesized. Such thinking can lead to new hypotheses, which can refine the existing theories or even eventually replace them entirely by new ones.

The word ‘modelling’ is used in ecological research in many different ways, ranging from mathematical models used to develop theoretical ecology to statistical models used to analyse empirical data. Mathematical modelling helps to understand the causal pathways from ecological mechanisms to the resulting phenomena, whereas statistical models are needed to confront theories and models with data. As a simple example, a theoretically minded ecologist might model resource–consumer dynamics with a system of differential equations (as we will do in Chapter 4). Insights from the model may be derived using just pen and paper, e.g. to show that a consumer population will persist in the long term if and only if the rate of resource production exceeds some threshold value, or to understand more generally how the dynamics of the consumer depends on the underlying ecological processes and their
parameter values. An empirically minded researcher could study the same system by acquiring experimental data on consumer abundance for different levels of resource availability. Statistical models may then be applied to show that in the experiment the growth rate of the consumer increases with the availability of resources, as suggested by the mathematical models constructed by the theoretical ecologist.

1.1 Forward and inverse approaches

One way of classifying different approaches to ecological modelling is that of ‘forward’ and ‘inverse’ approaches. Forward approaches are those where assumptions are made about the underlying mechanisms, and mathematical or simulation tools are used to study the consequences of those assumptions. With an inverse approach, the aim is to uncover the mechanisms that might have generated the patterns observable in empirical data. Both the forward (moving from mechanisms to patterns) and the inverse (moving from patterns to mechanisms) approaches involve their own challenges. To illustrate, let us consider a researcher who is interested in the movement ecology of butterflies, specifically on how the characteristics of the species and the structure of the environment influence the displacements that the individuals make over their lifetimes (as we will do in Chapter 2). If taking the forward approach, the researcher could track the detailed movement tracks of individuals to learn about the mechanisms that influence lifetime displacements, such as flight speed and directional persistence under different environmental conditions. Such data could be used to construct a movement model, which could be used to simulate lifetime movements in heterogeneous landscapes. While this can be relatively straightforward to do, the critical question is how much the model can be trusted. This is because the researcher is likely to obtain detailed movement data only for a limited number of individuals, only for a limited set of environmental conditions, and only over small temporal and spatial scales. This can be problematic, as the key features of movement behaviour may differ among individuals, environmental conditions, and temporal and spatial scales. More generally, a major challenge in the forward approach is that of extrapolation.

Conversely, if taking the inverse approach, the researcher could start by acquiring capture-mark-recapture data, i.e. marking and releasing butterflies when encountering them, and then recapturing some of the marked individuals later at another location. Such data may be acquired at the scale of lifetime displacements, and thus they would avoid the problem of extrapolation. However, it can be highly challenging to use such data for inference about the mechanisms that might have generated the observed displacements. This is because the data are simultaneously influenced by a myriad of processes, such as the intrinsic behaviour of the species, the spatial distribution of habitat characteristic, the prevailing weather conditions, and where and when the captures were attempted. Inverse problems are generally ill posed in the mathematical terminology, meaning that they do not have a unique solution. For example, assume that only few butterflies were recaptured. This may be because the
death rate was high or alternatively because the emigration rate out of the study area
was high, or because the capture rate was low.

Much of the current research in ecology and evolutionary biology uses both for-
ward and inverse approaches in an integrated manner. This is partly due to the
increasing interest in conducting ecological research at the interface between the-
ory and data, and partly due to developments in hardware and software that allow
for computationally demanding analyses. In this book, our emphasis is to combine
the two. To do so, we will use mathematical models to generate various kinds of data
sets. We will then apply statistical methods to those data, with the aim of asking how
much of the assumed mechanisms we can recover.

**I.2 The interplay between models and data**

Figure 1.1 illustrates the interplay between models and data in the context of the
forward approach. Often one wishes to understand how some phenomenon at a
higher level of biological organization emerges from the processes at a lower level
of biological organization. In the example of Figure 1.1, we are interested in how the
dynamics of a population emerge from the deaths and births of individuals. In the
process of model construction, knowledge about the ecological context is required.
This could involve e.g. whether the organism has discrete or overlapping generations,
and whether it inhabits a seasonal or unseasonal environment. These assumptions
translate into the model structure. In addition to deciding about the model struc-
ture, one needs to decide about the parameter values. In scenario simulations, the
parameter values are assumed rather than estimated, which is a perfectly valid ap-
proach if the aim is to ask how the parameter values influence the model’s behaviour.
To confront the model with data, the parameter values need to be estimated. In the
most pure form of the forward approach, the model parameters are simply measured
one by one. For example, one might monitor the lifetimes of individuals at different
population densities to measure the mean death rate and its dependency on popu-
lation density. Once the parameters have been measured, the model can be used to
generate predictions. To validate the model, the predictions are to be contrasted with
independent data. For example, one may compare the dynamics generated by the
model with data acquired directly at the population level. If the match between model
prediction and data is good, one may conclude that the model is compatible with the
data. This is, of course, encouraging, yet it does not mean that the model is correct, as
different kinds of models may end up with the same prediction. To refine the model
or to select between alternative types of models, new data, ideally from a different
context, are needed. Testing whether the model can predict such new data is called
cross-validation. If the model does not match well with the data, that is actually not
such a bad result. In this case, the conclusion is that the hypothesis that corresponds
to the model is not compatible with the data. Thus, something is wrong either with
the structural model assumptions, or with the parameter values. Through analysing
the mismatch between the model’s prediction and the data in more detail, one may
Figure 1.1 A flow diagram illustrating the interplay between models and data by the forward approach. We have assumed here that the model is constructed at the individual level, with the aim of predicting dynamics at the population level.

be able to infer reasons for the failed predictions, and thus learn more about the ecological context or the parameter values.

Applying the pure forward approach of Figure 1.1 can be challenging due to three problems. The first problem is that ecological processes are complex and influenced by manifold factors; thus, it is very difficult to derive the structural model assumptions from first principles. The second problem is that it is usually very difficult to perform direct measurements of all the model parameters. Even if this would be possible, the third problem is that the parameter estimates usually involve large amounts of uncertainty, and when a large number of uncertain estimates are put together through uncertain and nonlinear model structures, the amount of uncertainty becomes amplified.

Figure 1.2 presents an overview of the processes of model construction, parameterization, and inference, now performed in a manner that combines elements from both the forward and the inverse approaches. Like with the forward approach, the researcher uses prior knowledge about the ecological context to formulate a model, or a family of models, that describe the structural assumptions about the assumed underlying biology. While in the pure forward approach we assumed that all model parameters were measured directly, here we assume that direct information is possibly available only for some of the parameters, and that this information
1.2 The interplay between models and data

Figure 1.2 A flow diagram illustrating the interplay between models and data by combining elements from the forward and inverse approaches.

Parameter estimation should always be followed by model validation. To start with, it is always possible to compare model predictions to the data used to parameterize the model. If the model is not able to reproduce the data used to parameterize it, the problem is not in the parameter values but in the structural model assumptions, and thus in the assumptions made when describing the ecological context. If the model is able to reproduce the data used to parameterize it, it is encouraging, but does not necessarily mean that the mechanisms incorporated in the model actually operate in reality. To move one step forward, one may submit the model to a harder test, which is to make a prediction in a context that is different from the one used for parameter estimation. If the model is successful in predicting the new data as well, there is a stronger support that the assumptions behind the model reflect reality. If this is not the case, one may think about how to refine the model so that it simultaneously fits both the original data and the new data.